Baryon Number Non-Conservation and the Topology of Gauge Fields

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Abstract

An introduction to the subject of baryon number non-conservation in the electroweak theory at high temperatures or energies is followed by a summary of our discovery of an infinite surface of sphaleron-like configurations which play a key role in baryon-number non-conserving transitions in a hot electroweak plasma.

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1 Introduction

Any electroweak gauge theory built on SU(2)×U(1) with chiral fermions, such as the Standard Model, permits non-perturbative processes that do not conserve baryon number (B). The processes are associated with an energy barrier of height $E_{\text{sph}} \sim 4M_W/\alpha_{\text{w}} \approx 10$ TeV that suppresses the tunneling rate at zero energy and temperature with a factor $\exp(-4\pi/\alpha_{\text{w}}) \approx e^{-170}$. The B non-conserving transition rates become unsuppressed in a high-temperature electroweak plasma and possibly also in high-energy particle collisions[1].

Within an extended Standard Model the observed excess of matter versus antimatter in the universe may be explained as originating from B non-conserving processes in the hot electroweak phase transition ($T \gtrsim 100 - 300$ GeV). In multiparticle high-energy collisions with a center-of-mass energy E it has been shown perturbatively that the tunneling cross-section increases exponentially in $E^{4/3}$ for $E \ll E_{\text{sph}}$, but the perturbative analysis becomes unreliable for $E \lesssim E_{\text{sph}}$. Non-perturbative approaches to barrier tunneling at high-E collisions attract much interest and require further study.

Because of the chiral anomaly the baryon number is related to the Chern-Simons number N_{CS}, which characterizes the winding of the gauge fields, by the conservation law $\Delta(B - n_{\text{F}}N_{\text{CS}}) = 0$ where n_{F} is the number of families. B non-conservation is therefore accompanied by large, non-perturbative changes in the gauge fields corresponding to transitions between degenerate minima of the SU(2)×U(1) vacuum. The existence of an infinite number of such minima, labeled by an integer N_{CS}, is a consequence of the fact that the SU(2) field tensor W_{ij}^A (the square of which appears in the energy) becomes zero whenever $W_i \equiv W_i^A \sigma_A$ is of the pure-gauge form

$$W_i = -i(\partial_i U(\boldsymbol{x}))U^{-1}(\boldsymbol{x}). \tag{1}$$

Two adjacent minima $N_{\text{CS}} = n$ and $N_{\text{CS}} = n + 1$, $n \in \mathbf{Z}$, are separated by a high energy barrier. The highest point on the lowest possible path across the barrier corresponds to the energy E_{sph} and has been identified as the *sphaleron* solution [2]. It is a saddle point in configuration space with only two descending directions and Chern-Simons number

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$N_{\mathrm{CS}} = n + 1/2$.

2   Baryon number non-conservation in a hot electroweak plasma

In a hot electroweak plasma $\Delta B \neq 0$ processes have been thought to occur via classical thermal fluctuations across the barrier through the sphaleron saddle point. For temperatures $T \ll E_{\mathrm{sph}}$, the transition rate Γ is proportional to the Boltzmann factor $\exp(-E_{\mathrm{sph}}/T)$. For temperatures in the range $M_W \lesssim T \lesssim E_{\mathrm{sph}}$, Gaussian fluctuations about the sphaleron produce a prefactor[3] $M_W^4 (E_{\mathrm{sph}}/T)^3$. Because of the prefactor, the transition rate would decrease steeply at high temperatures[1]. This result is in contradiction with a scaling argument suggesting that $\Gamma \sim T^4$ in the symmetric phase (T being the only dimensionful parameter) as well as with lattice real-time simulations[5] that find $\Gamma = \kappa(\alpha_w T)^4$, $\kappa \approx 1$.

In a recent publication[6] we have shown the existence of an infinite "surface" of configurations with sphaleron-like properties, on which the sphaleron configuration is the point of lowest energy. These configurations mediate B non-conserving processes in the hot broken phase as well as in the symmetric phase. For temperatures $T \lesssim E_{\mathrm{sph}}$, the probability of thermal fluctuations with energy $E > E_{\mathrm{sph}}$ is considerable, and many classical paths other than those leading through the sphaleron are accessible and contribute to the transition rate. The perturbative expansion around the sphaleron is unreliable for $E \gtrsim E_{\mathrm{sph}}$ partly because, in our view, the multitude of sphaleron-like configurations with energy near E_{sph} is not properly accounted for in that approximation.

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[1]Recent calculations of bosonic and fermionic 1-loop corrections to the transition rate do not affect the prefactor; see Ref. 4 and references therein.

3 Infinite surface of sphaleron-like configurations

Sphaleron-like configurations[2] can be identified through the following observations about the sphaleron solution [2]: (i) its gauge field is odd, $W_i(\boldsymbol{x}) = -W_i(-\boldsymbol{x})$, and approaches the form (1) at infinity, where $U(\boldsymbol{x})$ is likewise odd, (ii) it has a half-integer Chern-Simons number $N_{\mathrm{CS}} = n + 1/2$, $n \in \mathbf{Z}$, (iii) the Dirac equation in the sphaleron gauge-field background has exactly one zero-mode. For this reason, one fermion energy level crosses zero in a transition through the sphaleron. In the Dirac sea picture, this is consistent with the creation or destruction of a baryon (anti-baryon). In fact, baryon number non-conservation requires, in addition to a change in Chern-Simons number, the crossing of an odd number of fermion energy levels.

The property (i) leads us to consider the class of *generalized odd* fields,

$$W_i(-\boldsymbol{x}) = -W_i^S(\boldsymbol{x}) \equiv -[S(\boldsymbol{x})W_i(\boldsymbol{x})S^{-1}(\boldsymbol{x}) + i\partial_i S(\boldsymbol{x})S^{-1}(\boldsymbol{x})], \qquad (2)$$

for some element S of the gauge group. This class of fields includes the odd fields but is much larger. The definition (2) of generalized oddness is gauge invariant. All results presented below have been obtained for generalized odd fields [6], but for simplicity we restrict to ordinary odd fields in this presentation.

From property (i) and Eq. (1) one easily derives that $\partial_i[U^{-1}(-\boldsymbol{x})U(\boldsymbol{x})] = 0$, and thus $U(\boldsymbol{x}) = \pm U(-\boldsymbol{x})$ as $|\boldsymbol{x}| \to \infty$. The class of odd fields therefore naturally splits into two disconnected classes, for which one can show the following: Fields with even U at infinity have integer N_{CS} and are continuously connected to a minimum of the vacuum through odd-parity fields. Fields with odd U at infinity have half-integer N_{CS} and are continuous deformations of the sphaleron within the class of odd-parity fields [3].

The Dirac equation in a background of odd-parity fields W_i with odd U can be shown to have an odd number of zero-modes. Therefore, a transition through any such sphaleron-like configuration will lead to the crossing of an odd number of fermion energy

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[2]For simplicity we present here only the case of SU(2) and vanishing Yukawa couplings. For more general cases, see Ref. 6 and references therein.

[3]These results have an independent proof within the theory of homotopy groups of maps [6].

levels. The proof is simple: Consider an eigenfunction $\psi(\boldsymbol{x})$ to the three-dimensional Dirac equation $\sigma_i(i\partial_i - W_i)\psi(\boldsymbol{x}) = E\psi(\boldsymbol{x})$ with energy $E \neq 0$. Then $\psi(-\boldsymbol{x})$ is an eigenfunction corresponding to the energy eigenvalue $-E$. Thus, non-zero eigenvalues are paired $(E, -E)$. Consider now a continuous deformation of the gauge field away from the sphaleron, in whose background field we know that the Dirac equation has one zero-mode. As the field is varied within the class of odd-parity fields, positive and negative eigenvalues will appear or disappear in pairs, and the number of zero-modes will stay odd.

Although there is no reason to believe that all B non-conserving processes occur in transitions through odd-parity odd-U fields, there is an argument which suggests that such configurations are energetically favored in thermal fluctuations near and above E_{sph}. Put in other words, the energy rises steeply to inaccessible values as one ascends from the sphaleron in all directions except along the odd-parity configurations. The simple reason is that the energy functional is invariant when $W_i(\boldsymbol{x}) \to -W_i(-\boldsymbol{x})$ for an arbitrary field W_i, and the odd-parity fields constitute a fixpoint under this transformation.

4 Conclusions

We find an infinite set of configurations other than the sphaleron which are the loci of fermion energy-level crossings in baryon number non-conserving thermal transitions. This result is independent of the Higgs sector and applies equally to the broken and the symmetric phases. The configurations, odd under a generalized parity, are easily excited thermally for $T \lesssim E_{\text{sph}}$ and play a key role in baryon number non-conserving transitions in a hot electroweak plasma. Their relevance in high-energy collisions remains to be investigated.

Acknowledgments

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